Learning to Think Mathematically with the Rekenrek

A Resource for Teachers, A Tool for Young Children

Authored by

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<u>Overview:</u> **This book provides teachers with the theoretical basis, practical knowledge, and expertise to use a powerful mathematical tool called the Rekenrek (also known as the Arithmetic Rack). Building on the idea that children must be able to "see" numbers within other numbers (e.g., 7 might be thought of as "5 and 2 more"), this book helps children recognize number combinations of five and ten, develop a rich sense of numbers between 0 – 20, and build a powerful set of intuitive strategies for addition and subtraction of both single and double digit numbers.**

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About the Author

Dr. Jeffrey Frykholm is an Associate Professor of Education at the University of Colorado at Boulder. A former public school mathematics teacher, Dr. Frykholm has spent the past 19 years teaching young children, working with beginning teachers in preservice teacher preparation courses, providing professional development support for practicing teachers, and working to improve mathematics education policy and practices across the globe (In the U.S., Africa, South America, Central America, and the Caribbean).

Dr. Frykholm also has authored over 30 articles in various math and science education journals for both practicing teachers, and educational researchers. He has been a part of research teams that have won in excess of six million dollars in grant funding to support research in mathematics education. He also has extensive experience in curriculum development, serving on the *NCTM Navigations* series writing team, and having authored two highly regarded curriculum programs: An integrated math and science, K-4 program entitled *Earth Systems Connections* (funded by NASA, 2005), and an innovative middle grades program entitled, *Inside Math* (Cambium Learning, 2009). *Learning to think Mathematically with the Rekenrek* is his third curriculum program, designed specifically for teachers in the elementary grades.

Dr. Frykholm was a recipient of the highly prestigious National Academy of Education Spencer Foundation Fellowship, as well as a Fulbright Fellowship in Santiago, Chile to teach and research in mathematics education.

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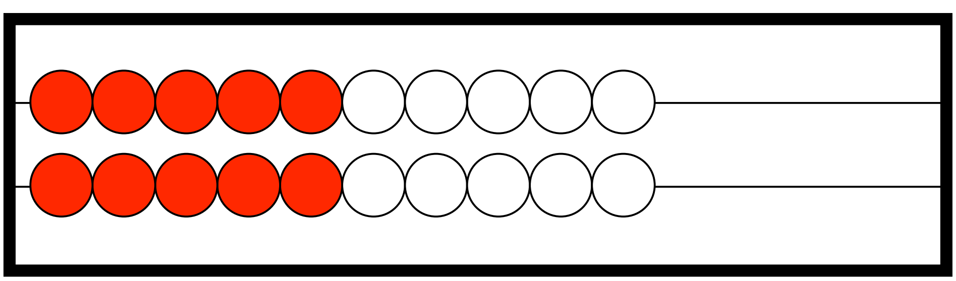
Chapter One: About the Rekenrek

There is perhaps no task of greater importance in the early grades than to help young children develop powerful understandings of numbers – their meanings, their relationships to one another, and how we operate with them. Mathematics educators have long focused on these objectives, doing so through the use of various models. Counters, number lines, base-10 blocks, and other manipulatives have been used for decades to cultivate number sense and beginning understandings of addition and subtraction. While each of these models has been shown to be effective in fostering mathematical reasoning, researchers agree that each of these models is limited.

More recently, the *Rekenrek* (also called an arithmetic rack) has emerged as perhaps the most powerful of all models for young learners. Developed by mathematics education researchers at the highly regarded Freudenthal Institute in the Netherlands, the Rekenrek combines various strengths inherent in the previously mentioned models in one compelling and accessible tool. The Rekenrek was designed to reflect the natural intuitions and informal strategies that young children bring to the study of numbers, addition, and subtraction. The Rekenrek provides a visual model that encourages young learners to build numbers in groups of five and ten, to use doubling and halving strategies, and to count-on from known relationships to solve addition and subtraction problems. With consistent use, over a short period of time children develop a rich sense of numbers, and intuitive strategies for solving problem contexts that require addition and subtraction.

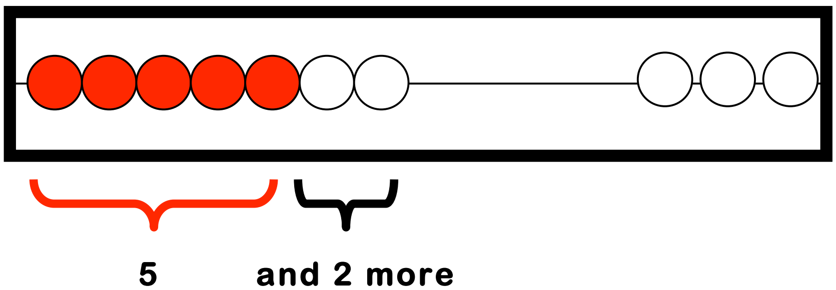
What is the Rekenrek?

As noted above, the Rekenrek combines features of the number line, counters, and base-10 models. It is comprised of two strings of ten beads each, strategically broken into two groups: five red beads, and five white beads. Readily apparent in this model is an implicit invitation for children to think in groups of five and ten.

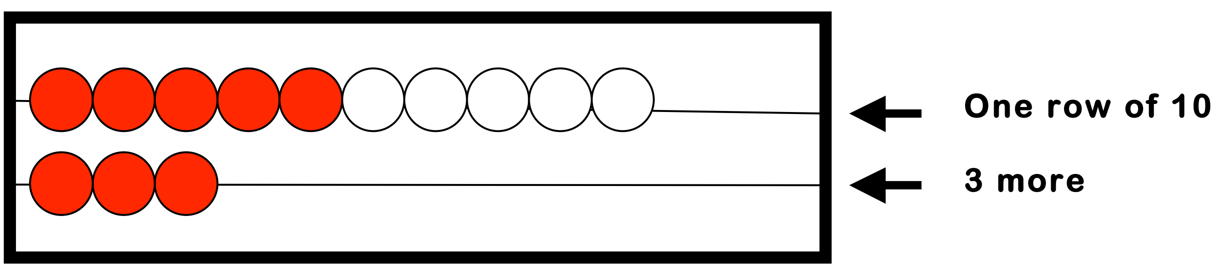


Of course, this model can also be adapted to accommodate children who may be either more or less advanced. One string of five or ten beads may be easily created, just as a teacher may wish to use two strings of twenty beads each. In any case, these alternative Rekenreks can be used to teach the same concepts for groups of children who may be at varying levels of cognitive development.

The structure of the Rekenrek – highlighting groups of 5 – offers visual pictures for young learners at the beginning stages of understanding that one number may be a combination of two or more other numbers. With the Rekenrek, children quickly learn to "see" in groups of 5 and 10. Therefore, the child will see the number 7 as two distinct parts: one group of 5, and two more. Likewise, the child sees 13 as one group of 10, and 3 more.



Seven is seen as **"5 and 2 more"**



Thirteen is seen as **"10 and 3 more"**

Research has consistently indicated the importance of helping children visualize number quantities as a collection of objects. Most adults, for example, do not need to count the individual dots on dice to know the value of each face. With similar intent in mind, using the Rekenrek with its inherent focus on 5 and 10 is instrumental in helping children visualize numbers, seeing them as collections of objects in groups. This strategy of seeing numbers "inside" other numbers – particularly 5 and 10 – is a precursor to the development of informal strategies for addition and subtraction that students will naturally acquire through repeated use with the Rekenrek. The activities in this book are sequenced to foster such development.

Chapter Two: A Rationale for the Rekenrek

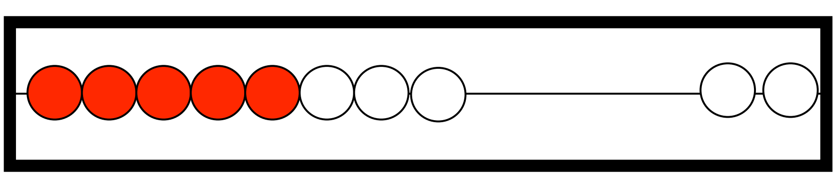
It is always important to have a rationale for the methods we employ when teaching mathematics to young children. Using manipulatives without a rich understanding of the potential – and the possible pitfalls – of the tools can not only limit their effectiveness, but in some cases interrupt the natural and desired development of mathematical thinking among young children. Hence, the following paragraphs are intended to provide you with a brief summary of the theory that underlies the use of the Rekenrek.

Cardinality

Typically, we convey the notion that numbers are synonymous with things to be counted. That is, we often stress the memorization of the number sequence as a sequential counting tool, not unlike the way in which children can recite the alphabet. While children do spend time counting in sequence – an important skill – we must also endeavor to help them develop *cardinality*, the recognition of a one-to-one correspondence between the number of objects in a set, and the numeral we use to denote that grouping of objects. Typically, children learn *how to count* before they understand that the last count word indicates the amount of the set – that is, the *cardinality* of the set. The Rekenrek helps young children to see numbers as groups (e.g., groups of 5, groups of 10, "doubles", etc.), rather than having to count every object in every set.

Subitizing

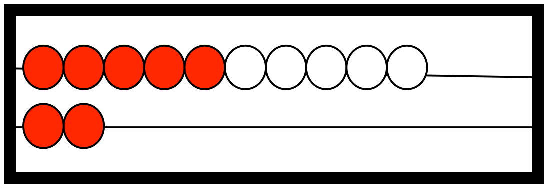
Educational psychologists and mathematics education researchers use the term *subitizing* as a construct used to describe the cognitive processes through which we recognize number patterns, and associate a numeral with a given quantity. While this construct can be rather complex depending on the way it is being elaborated, for the purposes of helping young children develop number sense through the use of the Rekenrek, we might best think of *subitizing* as the ability to instantly recognize particular groupings of objects *within a larger grouping* without having to count each individual element. The Rekenrek helps children build on their natural capacity to *subitize* in order to recognize quantities up to 10 (and beyond for more advanced students) without depending on the routine of counting. In the example below, the child uses the structure of the Rekenrek (5's and 10's) to *subitize*… to see 8 as a group of 5 and three more, without having to count any individual beads.

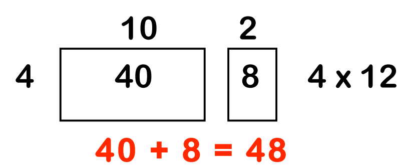


8 is *subitized,* and seen in two groups: One group of 5, and one group of 3.

Decomposition: Part, Part, Whole

Once children are able to subitize, it is only a matter of time before they will be able to do more complex *decomposition* of numbers, a concept that is essential for children to understand if they are to complete operations on numbers with meaning. Indeed, the two concepts are related. A student may subitize the number 12 as a group of 10, and 2 more. Some mathematics educators refer to this process as a *part, part, whole representation* – determining the individual parts that comprise the whole. Later, we may use this notion to *decompose* the number 12 in order to operate on it. For example, as shown at the right, 4 x 12 might be thought of as 4 x 10 plus 4 x 2 as illustrated in this area model. The Rekenrek is instrumental in setting the stage for conceptual understanding of decomposition as it relates particularly to the operations.



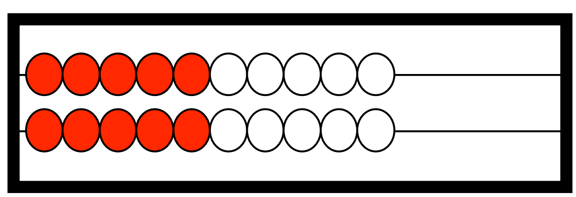


Anchoring in Groups of Five and Ten

The importance of helping children *group in 5's and 10's* cannot be emphasized enough. Two common manipulative models that are often used with great success are the 5-Frame and 10-Frame models. There are numerous and readily available activities for 5-frame and 10-frames that help children become comfortable with the notion of 10, the foundation upon which our entire number system is built. In some senses, the Rekenrek can be thought of as a *dynamic* 10-frame model. Since 10 does play such a large role in our numeration system, and because 10 may be found by combining two groups of 5, it is imperative that we help children develop powerful relationships for each number between 1-20 to the important *anchors of 5 and 10*.

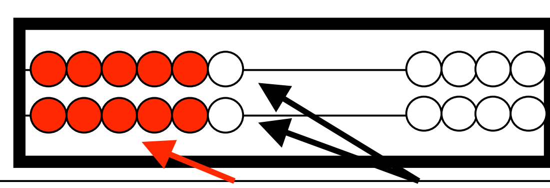
0	0	0	0	0
0	0			

10 frame



Informal Strategies: Doubling, Halving, One/Two More, One/Two Less

The Rekenrek can aid in the development of informal strategies for addition and subtraction that are essential for later work with larger two and three-digit numbers. Students learn strategies like *doubling and halving* (and the associated number facts, e.g., 6 + 6 = 12) as well as the notion of "adding on" by ones and twos. With the Rekenrek, children quickly associate numbers in relation to each other. For example, seven can be seen as "2 less than 9" or, "one more than 6."



6 + 6… 2 groups of 5 = 10, and 2 more = 12

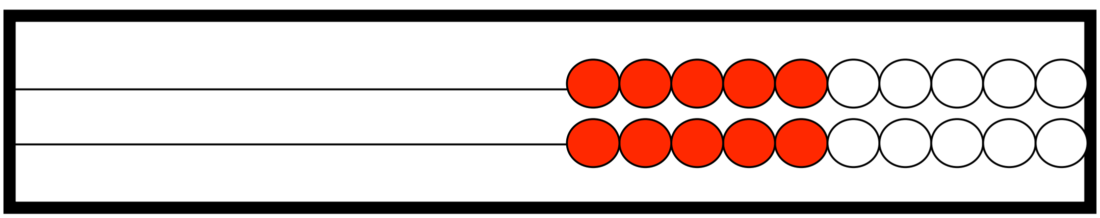
Chapter Three: Activities with the Rekenrek

How to Begin with the Rekenrek

The remainder of this book provides teachers with a sequence of activities that can be used to develop number sense and confidence with informal strategies for addition and subtraction with numbers up to 20. Lessons have been labeled (Levels 1-3) to roughly correspond to grade levels K-1, 1-3, and 2-5. Of course, with some adaptation, most lessons can be used appropriately across each of these grade levels. Prior to presenting the activities, however, it is important to understand a few key strategies when introducing and using the Rekenrek.

Starting Position

It is important to establish the norm that all problems begin with the beads on the right end of the strings. This convention is necessary to ensure that all students begin the problems with the same visualization. Of course, once involved in the problem context, children may make their own decisions about how to manipulate the beads. But the starting point should be emphasized uniformly, and students should learn that beads become "in play" as they are slid from left to right.

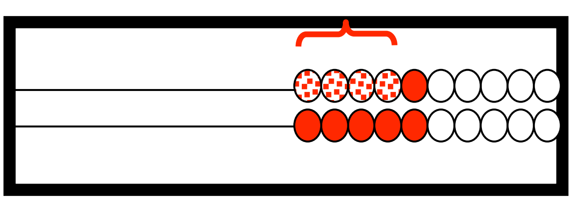


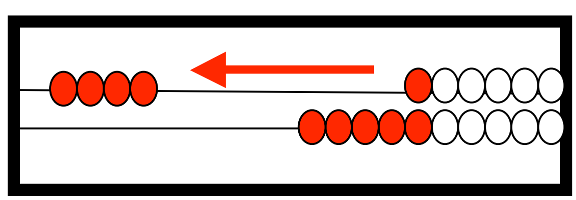
Starting Position for the Rekenrek

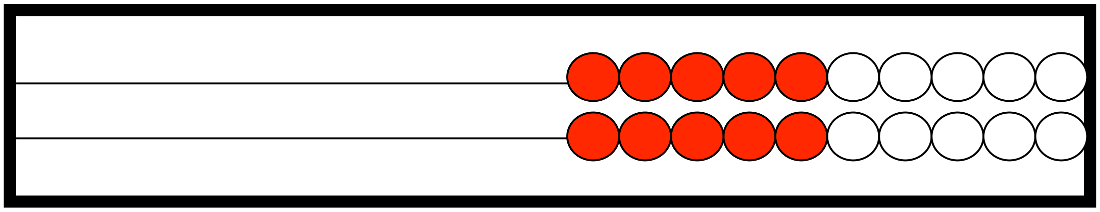
Manipulating the Rekenrek

One of the ideas that will be reinforced throughout the activities is that beads should be moved in clusters whenever it is possible for children to do so. In an effort to promote subitization, encourage children through your own modeling to slide beads to the left (and at times to the right) *in groups* rather than counting individual beads, and moving them one at a time. For example, a group of four should be slid to the left as one group rather than four individual beads. Model thinking strategies such as:

> **"Well, I need 4 beads to the left. Four is one less than 5. So, I do not need all five beads – I can leave one behind and slide 4 beads across."**







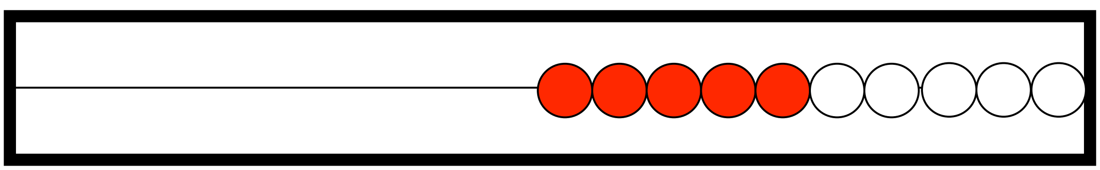
The Basic Rekenrek

Adapting the Rekenrek

Depending on the age and level of your students, you may wish to adapt the Rekenrek to be more suitable for a given group. With the youngest children, perhaps it is best to start with a Rekenrek that contains only 5 beads. This orientation is similar to the 5-frame manipulative mentioned previously, where number relationships between zero and 5 can be emphasized.

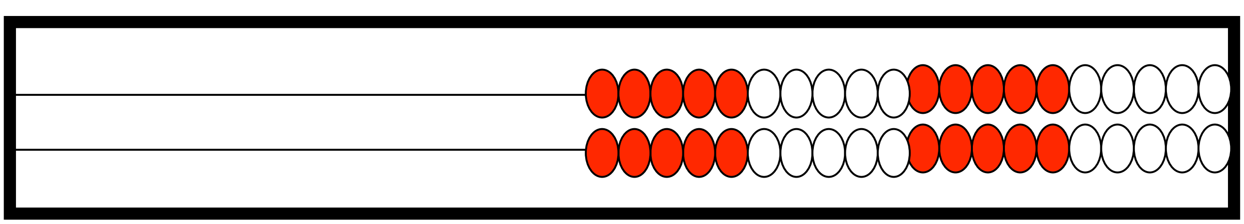
Rekenrek for the youngest children

Strings of ten beads can be used to strengthen understanding of number relationships between zero and 10, focusing in particular on quantities of 5 and 10. With a string of t10 beads, it is important to distinguish 5 and 10 by using different colors.



Rekenrek for emphasizing numbers between 0 and 10

For more advanced students who are ready to add and subtract multi-digit numbers, the Rekenrek can be extended to include 40 beads. Students should continue thinking in groups of 5 and 10, even as the numbers get larger.



Lesson 10: Almost a Double

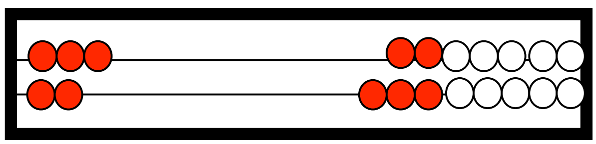
Lesson Level: TWO & THREE

Lesson Objectives

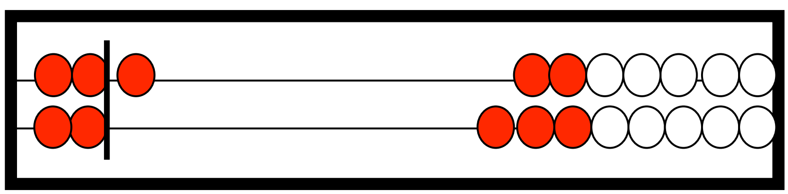
- To use an understanding of doubles relationships to work with near-doubles.
- To build understanding of the almost-double combinations (that are built upon the doubles anchors) to help solve computation problems.
- To recognize the difference between even numbers (can be represented as a pair of equal numbers) and odd numbers (paired numbers plus one).

Activity Background and Introduction

- Begin by illustrating the following on your demonstration Rekenrek.



- Ask students to find the double that is within the group of 5 beads on the left.
- Visually separate the double (two groups of two) from the remaining 5th bead. A pencil may be used to physically separate the beads.



- Ask students, "Now that you can see the double (i.e., 2 + 2 = 4), what can you say about a double plus one?"
- Illustrate the thinking by emphasizing the visualization, and also the symbolic equivalent:

2 + 3 = ? means…

↓ ↘

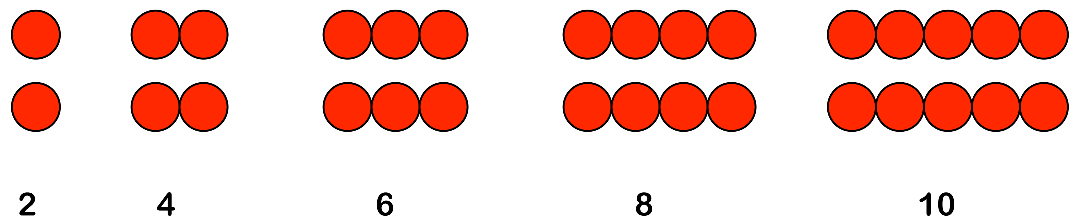
2 + (2 + 1) = ?

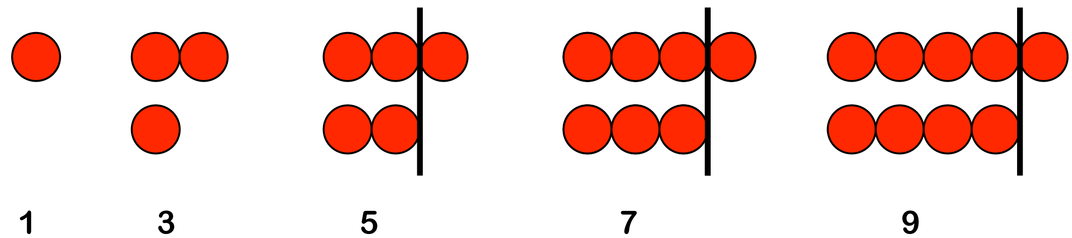
(2 + 2) + 1 = 4 + 1

Lesson Progression

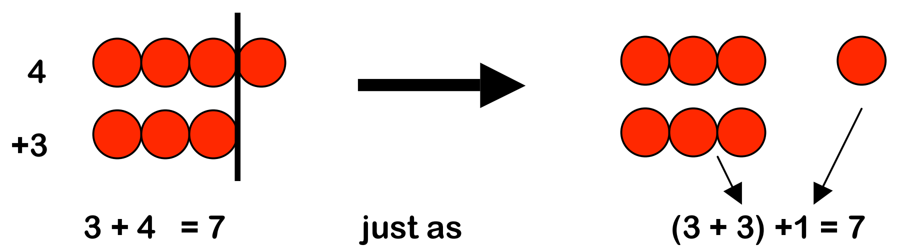
- Continue with additional examples. As students to find the double within the representation, and then use that information to find the sum of the near-double.
- Visually compare the doubles with the near doubles, highlighting the differences between even and odd numbers (odd numbers have solo bead).

Lesson 10: Almost a Double





- You may wish to highlight the doubles that may be found within these “almost double” visualizations as has been illustrated on the examples of 5, 7 and 9.
- Using the idea of hidden doubles as a way to work with “almost doubles” number relationships is a powerful strategy. It is important to help students transfer this visual strategy to symbolic representations. Take the example of 7. Children should be encouraged to see the “6” that exists within this starting number.



- By demonstrating this relationship, students begin to develop a relational view of the equal sign – that the equal sign means “the same as” rather than simply a symbol that indicates the answer is approaching.
- Develop this idea by doing several additional examples with the Rekenrek. Ask students to use the Rekenrek to “prove” whether or not the following are true. Have students visually identify each component of the statements.

  - Does 6 + 7 = 12 + 1?
  - Does 3 + 2 = 4 + 1?
  - Does 4 + 5 = 8 + 1?
  - Does 8 + 9 = 16 + 1?